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APPENDIX

A. Quasi-static Model of XRB

A cylindrical coordinate system ($r\phi z$) is used and rollers are classified based on their axial orientation: a "positive roller" if its axis points in the positive bearing axis direction, and a "negative roller" [23,24]. Fig. A1 shows the cross sections at the bottommost rollers. Also, an inclined coordinate system ($\xi\zeta\eta$) is introduced in Fig. A1(a), where the roller axis is inclined at angle κ to the bearing axis—also defined as the XRB contact angle (α). Displacement vector of the inner ring cross section is defined as:

$$\{u\} = \{u_r, u_z, \theta\}^T = [R\phi]\{\delta\} \quad \text{(A.1)}$$

$\{u\}$ denotes the displacement of the inner ring cross section, with two translational components, u_r and u_z, and a rotational component θ. The transformation matrix $[R\phi]$ is defined in [23]. The displacements of the inner ring cross section in the inclined coordinate system are given by:

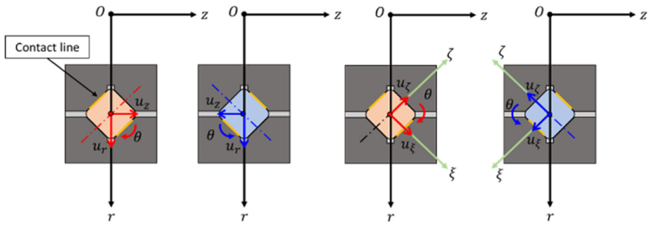
$$\{u_k\} = \{u_\xi, u_\zeta, \theta\}^T = [K]\{u\} \quad \text{(A.2)}$$

The transformation matrix $[K]$ is reported in [23]. Here, the axis is rotated equal to κ, which is also equal to the contact angle of XRB.

In addition to the inner ring displacement, the rollers also experience displacements and are defined as follows:

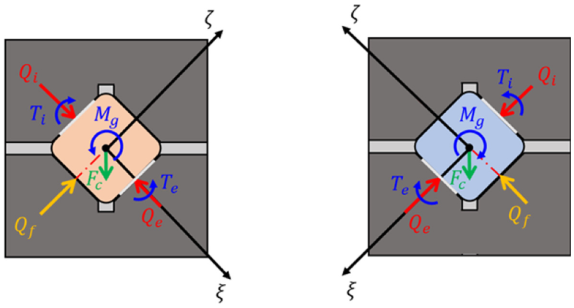
$$\{v\} = \{v_r, v_z, \psi\}^T, \{v_k\} = \{v_\xi, v_\zeta, \psi\}^T = [K]\{v\} \quad \text{(A.3)-(A.4)}$$

where $\{v\}$ is the roller displacement, which consists of two translation displacements and an angular displacement. Fig. A2 presents the free-body diagrams of the positive roller (Fig. A2(a)) and the negative roller (Fig. A2(b)).



(a) $r\phi z$ coordinate system (b) $\xi\zeta\eta$ coordinate system

Fig. A1 Local coordinate system



(a) Positive roller (b) Negative roller

Fig. A2 Free-body diagram of rollers

The contact load of one slice is determined by:

$$q_k = c_1 \delta_k^{\frac{10}{9}} \Delta l_k; \ (\delta_k > 0) \quad (k = 1,2,..., n_s) \quad \text{(A.5)}$$

Here, δ_k represents the contact compression between the roller and race at the kth slice, and n_s is the total number of slices along either the inner or outer race. The constant c_1 denotes the roller–race contact coefficient. Eq. (A.5) is then used to determine the reference rating life, as described in Eqs. (13)-(16). The race contact loads, roller-end contact load, and moments are calculated using:

$$Q = \sum_{k=1}^{n_s} q_k, \ Q_f = c_f \delta_f^{3/2}, \ T = \sum_{k=1}^{n_s} q_k l_k \quad \text{(A.6)-(A.8)}$$

Δl_k and l_k represent the length of a slice and the axial position of the kth slice. The roller end - outer race contact is assumed to be subjected to Hertzian, where δ_f represents the contact compression at the roller end- outer race contact, with c_f being the Hertzian contact constant. The roller profile used in this study is partial-logarithmic, defined as:

$$h_k = \begin{cases} 0.00045 D_a \ln\left[\dfrac{1}{1-\left(\dfrac{2|l_k| - l_m}{l_{eff}}\right)^2}\right], & \dfrac{l_m}{2} < |l_k| < \dfrac{l_{eff}}{2} \\ 0 & |l_k| \le \dfrac{l_m}{2} \end{cases} \quad \text{(A.9)}$$

l_m is defined as the flat middle length of the roller. For the race profile, it assumed to be purely crowned. The roller equilibrium in inclined coordinate system is given by:

$$\{F\} = \begin{Bmatrix} Q_i - Q_e + F_c\cos\kappa \\ Q_f - F_c\sin\kappa \\ T_i - T_e - M_g \end{Bmatrix} = \{0\} \quad \text{(A.10)}$$

By solving the roller equilibrium in Eq. (A.10), the roller displacement $\{v_\kappa\}$ and final roller contact loads can be found by conducting iterative Newton-Raphson method. After the equilibrium equations of all rollers and for both rows are successfully solved, the inner ring equilibrium can be obtained from the following equation:

$$\{F_b\} = \{F\} + \sum_{\tau=P}^{N}\sum_{j=1}^{Z} [R\phi]^T\{Q\}_j = \{0\} \quad \text{(A.11)}$$

The reactive force to the inner ring is defined as:

$$\{Q\}_j = [K]^T \begin{Bmatrix} -Q_i \\ 0 \\ -T_i \end{Bmatrix}_j \quad \text{(A.12)}$$

Eq. (A.11) is solved to determine the inner ring displacements with the given external load vector $\{F\}$ and then the iterative Newton-Raphson method is used again.

B. Rating Factor, b_m

The rating factor, b_m, is determined by extracting information from existing XRBs from a bearing manufacturer. Five large-sized XRBs with bore diameters 240, 300, 340, 400, and 500 mm were used as sample XRBs. As an initial guess, b_m, is set to equal to 1 and then the dynamic load rating is calculated. Since the basic dynamic load rating is a function of several parameters such as pitch diameter, roller diameter, roller effective length, and number of rollers per row, these parameters were used to create a curved-fitted equation based on the actual rating factor of the five XRBs which includes a clamping factor to maintain the values of b_m, be limited from 1 to 1.15.
Here, b_m can be estimated as:

$$b_m = 1.075 + 0.075\tanh[15\hat{b}_m(d_m, D_a, l_{eff}, Z) - c] \quad \text{(B.1)}$$

where, $\hat{b}_m(d_m, D_a, l_{eff}, Z)$ is a function of the bearing pitch diameter, roller diameter, effective length, and number of rollers per row which can be determined as:

$$\hat{b}_m(d_m, D_a, l_{eff}, Z) = e^X \quad \text{(B.2)}$$

The exponent X is defined by:

$$X = c_0 + c_1\ln d_m + c_2\ln D_a + c_3\ln l_{eff} + c_4\ln Z \quad \text{(B.3)}$$

where c_0 = –6.1567e–1, c_1 = 3.8079e–1, c_2 = –4.6971e–1, c_3 = 3.5962e–2, and c_4 = –9.6043e–2. Following the curve-fitting procedure, this rating factor is applied to the investigated XRB to estimate its basic dynamic load rating, ensuring alignment with the catalog values provided by the bearing manufacturer.

Table B1 Rating factor estimation

XRB designation	Axial dynamic load rating [N]		Error	b_m
	Catalog	ISO 281:2007	%	Estimated
SX011814	16,000	13,964	12.73	1.1457953
SX011848	149,000	134,534	9.71	1.1075267
SX011860	245,000	227778	7.03	1.0756087
SX011868	265,000	239225	9.73	1.1077438
SX011880	385,000	356,554	7.39	1.0797796
SX0118/500	550,000	508,872	7.48	1.0808219

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